**Метод Марквардта**

Методы оптимизации онлайн

Градиент:  
https://chart.googleapis.com/chart?cht=tx&chl=\frac%7b%20\partial%20f%7d%7b%20\partial%20x_%7b1%7d%7d%20=%202\cdot%20x_%7b1%7d-16  
https://chart.googleapis.com/chart?cht=tx&chl=\frac%7b%20\partial%20f%7d%7b%20\partial%20x_%7b2%7d%7d%20=%2012\cdot%20x_%7b2%7d%5e%7b3%7d-2\cdot%20x_%7b2%7d%2B14  
Матрица Гессе:

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| Hf = | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | https://chart.googleapis.com/chart?cht=tx&chl=\frac%7b%20\partial%5e%7b2%7df%7d%7b%20\partial%20x_%7b1%7d%5e%7b2%7d%7d | https://chart.googleapis.com/chart?cht=tx&chl=\frac%7b%20\partial%5e%7b2%7df%7d%7b%20\partial%20x_%7b1%7dx_%7b2%7d%7d | | https://chart.googleapis.com/chart?cht=tx&chl=\frac%7b%20\partial%5e%7b2%7df%7d%7b%20\partial%20x_%7b1%7dx_%7b2%7d%7d | https://chart.googleapis.com/chart?cht=tx&chl=\frac%7b%20\partial%5e%7b2%7df%7d%7b%20\partial%20x_%7b2%7d%5e%7b2%7d%7d | |  | |

**Итерация №1**.  
Значение градиента в точке x0: ▽f(x0) = (-0.01;17.71)  
Проверим критерий остановки:  
|▽f(X0)| = < ε  
Имеем:  
|▽f(X0)| = 17.7102>0.01  
Вычислим значение функции в начальной точке f(x0) = -37.9722. Сделаем шаг вдоль направления антиградиента  
S1 = -[H0+λ0I]-1▽f(x0)

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| S1 = -( | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 0 | | 0 | 18.684 | |  | | +1 | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 1 | 0 | | 0 | 1 | |  | | )-1 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -0.01 | | 17.71 | |  | | = (0.00333,-0.9) |

x1 = (7.995,0.758) + (0.00333,-0.9) = (7.998,-0.142)  
Вычислим значение функции в новой точке: f(x1) = -51.003  
Поскольку f(x1) < f(x0), то λ=1/2 = z∞  
**Итерация №2**.  
Значение градиента в точке x1: ▽f(x1) = (-0.00333;14.249)  
Проверим критерий остановки:  
|▽f(X1)| = 14.2493>0.01  
Вычислим значение функции в точке f(x1) = -51.0029. Сделаем шаг вдоль направления антиградиента  
S2 = -[H1+λ1I]-1▽f(x1)

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| S2 = -( | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 0 | | 0 | -1.277 | |  | | +z∞ | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 1 | 0 | | 0 | 1 | |  | | )-1 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -0.00333 | | 14.249 | |  | | = (0,0) |

x2 = (7.998,-0.142) + (0,0) = (7.998,-0.142)  
Вычислим значение функции в новой точке: f(x2) = -51.003  
Поскольку f(x2) < f(x1), то λ=z∞/2 = z∞  
**Итерация №3**.  
Значение градиента в точке x2: ▽f(x2) = (-0.00333;14.249)  
Проверим критерий остановки:  
|▽f(X2)| = 14.2493>0.01  
Вычислим значение функции в точке f(x2) = -51.0029. Сделаем шаг вдоль направления антиградиента  
S3 = -[H2+λ2I]-1▽f(x2)

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| S3 = -( | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 0 | | 0 | -1.277 | |  | | +z∞ | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 1 | 0 | | 0 | 1 | |  | | )-1 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -0.00333 | | 14.249 | |  | | = (0,0) |

x3 = (7.998,-0.142) + (0,0) = (7.998,-0.142)  
Вычислим значение функции в новой точке: f(x3) = -51.003  
Поскольку f(x3) < f(x2), то λ=z∞/2 = z∞  
**Итерация №4**.  
Значение градиента в точке x3: ▽f(x3) = (-0.00333;14.249)  
Проверим критерий остановки:  
|▽f(X3)| = 14.2493>0.01  
Вычислим значение функции в точке f(x3) = -51.0029. Сделаем шаг вдоль направления антиградиента  
S4 = -[H3+λ3I]-1▽f(x3)

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| S4 = -( | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 2 | 0 | | 0 | -1.277 | |  | | +z∞ | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  |  | | --- | --- | | 1 | 0 | | 0 | 1 | |  | | )-1 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -0.00333 | | 14.249 | |  | | = (0,0) |

x4 = (7.998,-0.142) + (0,0) = (7.998,-0.142)  
Вычислим значение функции в новой точке: f(x4) = -51.003  
Поскольку f(x4) < f(x3), то λ=z∞/2 = z∞